DR RICHARD PRICE, D.D. F.R.S.

'NO TAXATION WITHOUT REPRESENTATION'

Dr Richard Price – Financial World

- It was after his marriage that Richard Price first turned to those mathematical and philosophical studies which introduced him to a wide circle of acquaintances. He believed his duty and purpose in life was to be a minister and a preacher, and he believed that any activity not related to the ministry was trivial

- In 1761 he was called upon to examine papers of a deceased friend, Thomas Bayes. Bayes was a Presbyterian Minister and noted Mathematician

Dr Richard Price – Financial World

- The papers included an unsolved problem relating to probability. The question revolved around trying to determine the odds of something happening again if it or something else had happened or occurred

- Bayes was trying to determine what was the likelihood of God existing and on whether it was likely, indeed probable that miracles had taken place during Jesus' time on earth

Dr Richard Price – Financial World

- He edited Bayes' major work "An Essay towards Solving a Problem in the Doctrine of Chances" (1763) which contains Bayes Theorem, one of the fundamental results of probability theory

- They contained some remarks on a Friendly Society which led him to make calculations on the subject of Life Assurance

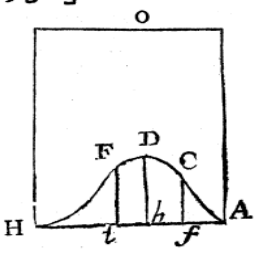
- Price wrote an introduction to the paper which provides some of the philosophical basis of Bayesian statistics

An Essay towards Solving a Problem in the Doctrine of Chances, 1763

LII. *An Eſſay towards ſolving a Problem in the Doctrine of Chances. By the late Rev. Mr.* Bayes, *F. R. S. communicated by Mr.* Price, *in a Letter to* John Canton, *A. M. F. R. S.*

Dear Sir,

Read Dec. 23, 1763. I Now ſend you an eſſay which I have found among the papers of our deceaſed friend Mr. Bayes, and which, in my opinion, has great merit, and well deſerves to be preſerved. Experimental philoſophy, you will find, is nearly intereſted in the ſubject of it; and on this account there ſeems to be particular reaſon for thinking that a communication of it to the Royal Society cannot be improper.



Now, in order to reduce the foregoing rule to practice, we muſt find the value of the area of the figure deſcribed and the ſeveral parts of it ſeparated, by ordinates perpendicular to its baſe. For which purpoſe, ſuppoſe A H = 1 and H O the ſquare upon A H likewiſe = 1, and Cf will be = y, and Af = x, and Hf = r, becauſe y, x and r denote the ratios of Cf, Af, and Hf reſpectively to A H. And by the equation of the curve $y = x^p r^q$ and (becauſe Af + fH = A H) $r + x = 1$. Wherefore $y = x^p \times \overline{1-x}^q = x^p - qx^{p+1} + q \times \frac{q-1}{2} \times x^{p+2} - q \times \frac{q-1}{2} \times \frac{q-2}{3} \times x^{p+3}$ + &c. Now the abſciſſe being x and the ordinate x^p the correſpondent area is $\frac{x^{p+1}}{p+1}$ (by prop. 10. caſ. 1. Quadrat. Newt.) * and the ordinate being qx^{p+1} the area is $\frac{qx^{p+2}}{p+2}$; and in like manner

* Tis very evident here, without having recourſe to Sir Iſaac Newton, that the fluxion of the area A Cf being $y\dot{x} = x^p\dot{x} - qx^{p+1}\dot{x} + q \times \frac{q-1}{2} x^{p+2}\dot{x}$ &c. the fluent or area itſelf is $\frac{x^{p+1}}{p+1} - q \times \frac{x^{p+2}}{p+2} + q \times \frac{q-1}{2} \times \frac{x^{p+3}}{p+3}$ &c.

Fellow of the Royal Society

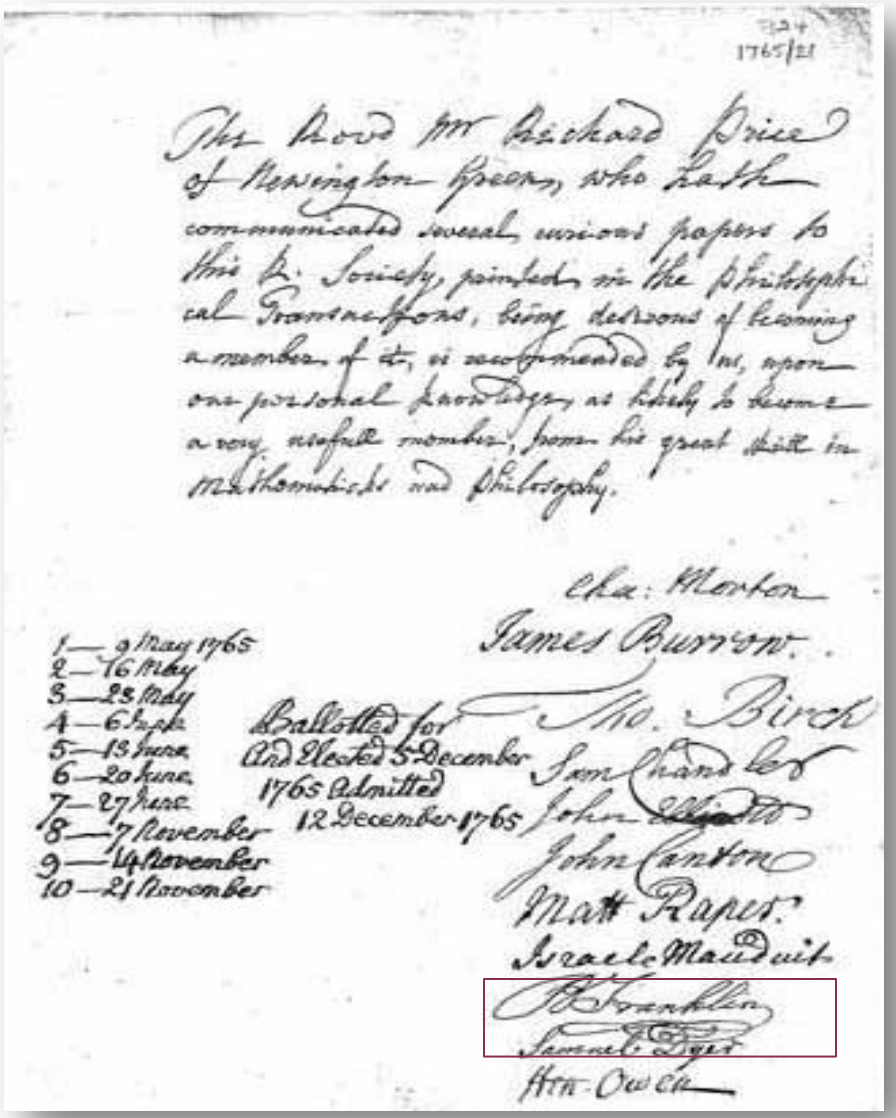
- Price communicated his finished paper to the Royal Society via John Canton, another visionary scientist of the day and a few days later Richard Price was sponsored to become elected to the Royal Society with one of the sponsors being his friend Benjamin Franklin.

- In 1765 he was elected a Fellow of the Royal Society in recognition of his work

Fellow of the Royal Society

For his work in mathematics and philosophy Price was elected to the Royal Society in 1765. One of his sponsors was his close friend Benjamin Franklin

Price also introduced Joseph Priestley to the society and served on its Council



Life Assurance

- At this time the public remained unconvinced as to the merits of Life Assurance, objecting that it was not based on scientific principles
- Richard Price was a good mathematician and began the task of proving otherwise, collecting statistical information in order to demonstrate the theory
- The Northampton Tables, for which he was responsible, were the result of an enquiry into the value of life in England at that time

The Northampton Tables

- In 1761 Thomas Bayes died. Bayes was a Presbyterian Minister and noted Mathematician. On his death his wife gave Richard Price his papers which included an unsolved problem related to probability

- The question revolved around trying to determine the odds of something happening again if it or something else had happened or occurred. For example, imagine you threw a set of dice (or die) and you got a double six. What Bayes wanted to know is what is the likelihood of you throwing another double six on your second throw?

The Actuary
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ACTUARIAL EDUCATION

by James J. Murphy

Accompanying this issue is a letter from President Lautzenheiser introducing an attached statement, *Strategic Premise for Actuarial Education*. Developed by Michael J. Cowell, 1981-82 General Chairman of the E & E Committee, its concepts, although not previously assembled into one document, have been guiding the Society's education efforts for decades. It will now provide useful background for consistent and organized changes in our education system.

While it was being written, several Education Task Forces have been preparing recommendations for changes in our syllabus. Articles by two of these Task Forces that have completed their work are printed here, viz.:

Operations Research and Applied Statistics

Mathematical Aspects of Demography

As other Task Forces complete their assignments, we will present similar reports in *The Actuary*. Also, as new Task Forces are established, we will announce them and will call for ideas and volunteers. Keep your eyes open for more news from E & E!

Operations Research and Applied Statistics, (James A. Tilley, Chmn.)

Formed in September 1981, with members drawn from both the academic and business communities, and with representation from the Canadian Institute of Actuaries and the Casualty Actuarial Society, this Task Force began by establishing criteria to guide us to decisions on syllabus and course of reading. Our starting premise was that the scope of actuarial work will continue to broaden to embrace all aspects of the financial products and services business; hence, actuaries should become familiar with

(*Continued on page 8*)

TWO CENTURIES AGO, THE NORTHAMPTON TABLE

To mark this year's bicentennial of Richard Price's Northampton Table—the earliest mortality table to be constructed especially for life insurance premiums—we show here the contrast of its life expectancies with those of population tables of, respectively, one and two centuries later.

Complete Expectations of Life

Age	*Northampton Table (Data of 1735-1780)*	*English Life Table No. 4 (Data of 1871-1880)*	*Calendar Year 1900 U.S. Life Tables **
0	25 yrs.**	43 yrs.	74 yrs.
20	35	41	55
35	26	30	41
50	18	20	28
65	11	11	16
80	5	5	8

*from *Actuarial Study No. 87*, Joseph F. Faber, Social Security Administration, Sept. 1982.
**q_0 in the Northampton Table was .258!

Since the two later of the above tables are sex-distinct, these figures show for them the arithmetic means of the $\mathring{e}_x$ values for males and females.

Notes on the Northampton Table

Actuary Price, well experienced by having studied mortality in other English towns and aware that he was aiming to generalize from a tiny base (a single church parish), evidently felt free to take major liberties with his data. The many adjustments he made were not in the values of q_x but in the column of deaths. For example, he showed exactly 75 deaths at every age from 21 to 39, 82 deaths each year from age 51 to 61, and 80 deaths from 64 to 75, in a table whose radix was 11,650 at age 0.

The improvement in longevity between the Northampton Table and English Life Table No. 4 is attributable in part to acknowledged overstatement of mortality in the former, and in part to progress in sanitation and medicine. Vaccination was introduced into England by physician Edward Jenner in 1796.

The Northampton Table, though by no means the earliest—Halley's Breslau table dates from 1693 and Kerrsboom's in Holland from 1738—is bound up with the history of the Equitable Life Assurance Society (of London). Griffith Davies, in his *Treatise on Annuities* (1825) quotes Price's nephew, William Morgan, thus:

> "(T)he Society had computed all their premiums from the (early 18th century) London Table of Observations (but, after seeing their experience from 1768 to 1780) they determined to compute the premiums in future from a table which should give the probability of life *higher* (emphasis in original) than that which they had hitherto used; and for this purpose they adopted one which had been just formed by Dr. Price, from very accurate observations made in the town of Northampton."

(*Continued on page 5*)

(Continued from page 4)

PRE-TEFRA

S^* = the smaller of:
(a) S, or
(b) [(G' − T), if positive,] + L.

TEFRA

S^* = the smaller of:
(a) S, or
(b) the larger of
(1) [(G' − T), if positive,] + L, or
(2) Q + the smaller of (i) (1+f) • (P+N), or
(ii) L+f • (P+N).

where L is $250,000 pre-TEFRA and is now defined as $1 Million, reduced for S > $4 Million (to 0 when S = $8 Million), allocated proportionately to the number of companies in the affiliated group. And where f = .85 for stock companies and f = .775 for mutual companies.

Thus, a company's tax position can be classified in terms of the amount of Special Deductions allowed under Section 809(f). Assuming increasing levels of S^*, the classes for stock companies would be:

Category V : $S^* = L$
Category W : $S^* = Q + 1.85\ (P + N)$
$L > (P+N)$
Category X : $S^* = Q + L + .85\ (P + N)$
$L < (P+N)$
Category Y : $S^* = G' - T + L$
Category Z : $S^* = S$

The variable L introduces a factor into the tax calculation that may come from data not included in the company's tax return. Also, a new corridor situation develops when an affiliated group's total special deductions fall in the range from $4 Million to $8 Million. Interesting marginal tax rates develop within this corridor.

Comments are being made indicating a switch of the tax phase for most mutual companies from Phase I to Phase II--. While being basically true, the statement is not fully accurate in that only a few companies will find themselves in the old Phase II-position. It might better be said that the old Phase II-companies, which previously had a $250,000 limit on Special Deductions, will join the old Phase I companies in a new category, both having a variable amount of allowable Special Deductions. □

The Northampton Table

(Continued from page 1)

It is right for us today to remember and to acclaim Richard Price's work. James S. Elston, in the second edition of *Sources And Characteristics of the Principal Mortality Tables* (1932) gives this endorsement by an 1823 author:

> "Dr. Price did as much as the nature of his materials would allow. For in those days no census or enumeration of the population had been made; and without (that) . . . an accurate Table of Observations cannot possibly be obtained."

My thanks to Howard W. Johnson, F.I.A. of London's Equitable Society for sending helpful material used in this account.

Wigglesworth's Table (1789)

"The first American table used at all for calculating life contingencies"—these words are quoted from *TASA* VII (1901), 3—made up from records in healthy portions of Massachusetts, was published, by Prof. Edward Wigglesworth of Harvard University, only six years later than was the Northampton Table.

E.J.M.

Golden Anniversary

(Continued from page 4)

remain. They became Fellows in the year in which the total number of Fellows went past the 400-mark; happily, 90 of those 400 are still with us.

The number of Associates who have 50 or more years as such is now 33.

The Society member who has been one for the longest time is Horace Holmes (F.S.A. 1921); he earned his Associateship in 1913 and is our only living member whose name is in the first published Index to the *Transactions* (1889-1914). Erston Marshall, though, is still our dean among Fellows, dating from 1919. □

THE PROPOSED NOTATION OF ENGELFRIET AND KOOL

by Frank G. Reynolds

(This is Article No. 6 in a series.)

Engelfriet and Kool explored the possibilities of using a linear form involving only the keys found on the standard typewriter keyboard. To replace the superscripts and lower left corner resort was made to an ingenious series of combinations of the special characters. For example, the double quotation symbol replaced the dieresis; ≍ was used to indicate that annuity payments were deferred for a given period and then continued, and this for a limited period from the end of the deferment period; the apostrophe was used to indicate that the annuity was payable in advance. Thus, $_{n|m}\ddot{a}_x^{(b)}$ became $''a \asymp (x,n,m,b)$.

For a compound status an additional letter was added to the stem to indicate last survivor and other conditions. In general, the proposal met its design criterion of being linear, of being readily transformable into programming names, and of using only typewriter characters. The problem was the extensive use of backspacing to create characters such as ≍ and the use of auxiliary symbols which made it difficult to relate symbols to the present notation. □

EXAM PREPARATION STUDY MANUALS

Study manuals for actuarial exams, a continuation of the series begun at Northeastern University in 1972, are available for all Spring 1983 exams *except* Part 10. Enquire from ACTEX, Box 2392, Framingham, MA 01701.

Richard L. London

His Financial World

- In time, Richard Price had become a great authority on the subject of Life Assurance. In its infancy the small business was carried on in a modest premises near Blackfriars Bridge
- Dr Price rode there almost daily from Newington Green where he lived. He rode on a white horse dressed in a blue greatcoat and black spatterdashers
- “There goes Dr Price, make way for Dr Price!” was often heard

Dr William Morgan

- William Morgan, Dr Price's nephew, moved to London and lived with his uncle for a time
- His uncle asked William – "Billy, do you know anything of mathematics?" – William replied "No uncle, but I can learn"
- In a very short time William Morgan made himself proficient, becoming the next actuary following the death of the Actuary of the Equitable Assurance Company in 1775

Equitable Life Assurance Society

- In due course Dr Price's nephew and his sister Nancy set up house together at Chatham Place where they remained for the next six or seven years

- William held the post of actuary for 55 years and, on his retirement, his son Arthur took over for a further 40 years. During this time the society became one of the largest financial institutions in the world. The Society ran into trouble in the first decade of the 21st century after ignoring Dr Price's advice which had worked so well over the previous two centuries

Dr Richard Price – William Pitt and National Debt

- Finance was the subject of most of Richard Price's important scientific papers. His opinion was sought after by leading statesmen
- He was well known for his tracts on the mounting British national debt. His proposals, first articulated in his 1769 "Observations on Expectation of Lives" was to set up a 'sinking fund' to retire public debt
- He persuaded the prime minister William Pitt the Younger to establish this fund in 1786 reallocating government revenues for the paying off of the debt

Dr Richard Price – William Pitt and National Debt

- Three plans for the extinction of the national debt were put forward by Price, the first of which was recommended by him. Pitt however chose the third, the least advantageous plan which wasn't carried out as Dr Price intended it

- Pitt chose to set aside £1 million every year to be used by national debit commissioners to purchase national debt and use the interest from the treasury to expand its purchases of more debt. The use of interest to pay for debt was Price's principal contribution

Dr Richard Price – William Pitt and National Debt

- By using the dividends on government stock to pay off debt in the following year, it was intended that eventually the debt would be abolished

- But at the end of Napoleonic wars the national debt was £16 million greater than it would have been without the sinking fund because of the obligation to purchase stock at low interest rates during the war with funds borrowed at high rates of interest in the market. The sinking fund was phased out during the 19th century

Dr Richard Price – His Financial Legacy

- What is truly astounding is the fact that Richard Price is not as well-known as he should be for his contribution to the foundations of the modern world. Without Price's works on mathematics which founded Actuarial Science we would not have had a stable and reliable Insurance Industry that enabled the underwriting the great engineering advances over the following two hundred and fifty years

- Everything from the Titanic to the Space Shuttle, from Macro-economics to Life Pensions are based on the probability theory of Richard Price. The cutting-edge modern science of creating Artificial Intelligence also relies on the work of this eighteenth century polymath